B.Sc. (Part-I) (Semester–I) (CBCS) Examination

MATHEMATICS

(I) Algebra & Trigonometry

Time : 3 Hours] [Maximum Marks : 60

Note :— (1) Question No. 1 is compulsory. Attempt once.

(2) Attempt one question from each unit.

1. Choose the correct Alternative :

(i) The matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is ______.

(a) Scalar matrix (b) Identity matrix

(c) Row matrix (d) Column matrix

(ii) A square matrix $A = [a_{ij}]$ is said to be skew-symmetric if :

(a) $a_{ij} = a_{ji}$ (b) $a_{ij} = -a_{ji}$

(c) Both (a) and (b) (d) None of these

(iii) The rank of zero matrix is :

(a) 1 (b) 0

(c) n (d) None of these

(iv) For a symmetric matrix the eigen vectors are :

(a) Equal (b) Orthogonal

(c) Parallel (d) None of these

(v) If α, β, γ are roots of $ax^3 + bx^2 + cx + d = 0$ then $\sum\alpha$ is _____.

(a) $\frac{b}{a}$ (b) $-\frac{b}{a}$

(c) $\frac{c}{a}$ (d) $\frac{d}{a}$

(vi) The number of positive and negative roots of an equation of degree n is found by :

(a) Carden's Method (b) Ferrari's Method

(c) Descartes' rules of signs (d) None of these

(vii) Identify the value of $\sin^{-1}x$

(a) $\log\left[x + \sqrt{x^2 + 1}\right]$ (b) $\log\left[x + \sqrt{x^2 - 1}\right]$

(c) $\log\left[x + \sqrt{1 - x^2}\right]$ (d) None of these

(viii) The value of $e^{-\frac{p}{2}i}$ is ______.

(a) $-i$ (b) $1 + i$

(c) $1 - i$ (d) 0

(ix) The series $\frac{p}{4} = \frac{1}{2} - \frac{1}{3} \times \frac{1}{2^3} + \frac{1}{5} \times \frac{1}{2^5} - \ldots\ldots\ldots + \frac{1}{3} - \frac{1}{3} \times \frac{1}{3^3} + \frac{1}{5} \times \frac{1}{3^5}$

is called as

(a) Rutherford's Series (b) Geometric Series

(c) Gregory's Series (d) Euler's Series

(x) The sum of infinite Geometric Series :

$a + ar + ar^2 + \ldots\ldots\ldots + a \,.\, r^{n-1} + \ldots\ldots, |r| < 1$ is

(a) $\frac{r}{a - r}$ (b) 1

(c) $\frac{a}{1 - r}$ (d) $\frac{r}{1 - r}$ 10×1=10

UNIT—I

2\. (a) Define Hermition matrix and show that $\begin{bmatrix} 2 & 4-i & 6i \\ 4+i & 1 & 3 \\ -6i & 3 & 0 \end{bmatrix}$ is hermitian matrix. 3

(b) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$ then verify $(AB)^1 = B^1 \,.\, A^1$ 2

(c) Find the adjoint of matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ -2 & 6 & 1 \end{bmatrix}$ 5

3\. (p) Prove that Every square matrix can be expressed as the sum of symmetric and skew-symmetric matrices. 2

(q) Let $A = \begin{bmatrix} 2 & 1 & 3 & -1 \\ 4 & 2 & 1 & -4 \\ 3 & -1 & 2 & 1 \end{bmatrix}$ then show that : 3

(i) $R_{23}^{-1} = R_{23}$

(ii) $R_1^{-1}(3) = R_1\left(\frac{1}{3}\right)$ and

(iii) $R_{21}^{-1}(-2) = R_{21}(2)$

(r) Reduce the matrix $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ to the normal form. 5

UNIT—II

4. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 4

(b) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$. 6

5. (p) Find the row rank and the column rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$. 4

(q) State Cayley-Hamilton theorem. Explain it for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. 6

UNIT—III

6. (a) State Descartes' rule of sign. Find the nature of roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$. 1+3

(b) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of

(i) $\sum\alpha^2$

(ii) $\sum\alpha^2\beta$

(iii) $\sum\alpha^2\beta^2$ 2+2+2

7. (p) Find the equation whose roots are the roots of $x^5 + 7x^4 + 7x^3 - 8x^2 + x + 1 = 0$ with their signs changed. 2

(q) Find the equation whose roots are the roots of $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each diminished by 4. 3

(r) Solve $x^3 - 15x^2 - 33x + 847 = 0$ by Cardon's method. 5

UNIT–IV

8. (a) Prove that $\frac{1 + \sin ? + i \cos ?}{1 + \sin ? - i \cos ?} = \sin ? + i \cos ?$.

Hence prove that $\left(1 + \sin \frac{p}{5} + i \cos \frac{p}{5}\right)^5 + i\left(1 + \sin \frac{p}{5} - i \cos \frac{p}{5}\right)^5 = 0.$ 3+3

(b) Find all the values of $(-i)^{1/6}$. 4

9. (p) If $\tan (A + iB) = x + iy$, then prove that $\tan 2A = \frac{2x}{1 - x^2 - y^2}$ and $\tan 2B = \frac{2y}{1 + x^2 + y^2}$.

Also show that $x^2 + y^2 + 2x\cot 2A = 1$ and $x^2 + y^2 - 2y \coth 2B + 1 = 0$. 2+2+2

(q) Separate into real and imaginary parts of $\cos^{-1}\left(\frac{3i}{4}\right)$. 4

UNIT—V

10. (a) Prove that :

$$\frac{p}{4} = \frac{1}{2} - \frac{1}{3} \cdot \frac{1}{2^3} + \frac{1}{5} - \frac{1}{2^5} - ...... + \frac{1}{3} - \frac{1}{3} \times \frac{1}{3^3} + \frac{1}{5} \cdot \frac{1}{3^5}.$$ 4

(b) Sum the series $\sinh x + n \sinh 2x + \frac{n(n-1)}{1.2} \cdot \sinh 3x + ...$ to $n + 1$ terms, where n is a positive integer. 6

11. (p) Find the sum of the series : $a \sin x - \frac{1}{3} a^3 \sin 3x + \frac{1}{5} a^5 \sin 5x + ...$ 6

(q) Prove that $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} = \frac{p}{4}$. 4